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$$\tan(\psi-\psi')=\frac{c'l-cl'}{cc'+ll'}\text{; and }\tan\tfrac{1}{2}(\psi-\psi')=\frac{\sqrt{(c^2+l^2)(c'^2+l'^2)}-cc'-ll'}{c'l-cl'}.$$

The tension in the chain at any point being equal to the weight of as much of it as would reach from that point to the directrix, and the resultant tensions in the two catenaries at the pegs being the same, it follows that the two catenaries have a common directrix. Also, the square of the ordinate of any point equals the sum of the squares of the parameter and the length of the arc between the vertex and that point. From these facts, $l^2+c^2=l'^2+c'^2$. Substituting,

$$\tan\tfrac{1}{2}(\psi-\psi')=\frac{c^2+l^2-cc'-ll'}{c'l-cl'}=\frac{(c-c')c+(l-l')l}{c'l-cl'}.$$

Remembering that $(c-c')(c+c')=(l'-l)(l+l')$, this may be reduced to $(c'-c)/(l+l')$.

But $c'-c$ is the distance between the vertices of the two catenaries, $l+l'$ is half the length of the chain, and $\psi-\psi'$ is the inclination of the portions near the peg. Hence the proposition is established.

Also solved by *G. B. M. ZERR.*

DIOPHANTINE ANALYSIS.

57. Proposed by JOSIAH H. DRUMMOND, LL. D., Counselor at Law, Portland, Maine.

Each of *five* of the digits may be the terminal figure of a perfect integral square. Each of *eighteen* combinations of two digits may be the *two* terminal figures of an integral square. Each of *one hundred and nineteen* combinations of three digits may be the *three* terminal figures of an integral square. *Under these conditions,* what is the greatest number of arrangements of the nine digits, all taken together, whose three terminal figures shall be those of a square number?

Solution by the PROPOSER.

The first two conditions are embraced in the third and must be disregarded: the terminal figure of the last three must be one of five digits; and the two terminal figures must be one of the eighteen combinations of two digits. There are 119 combinations of three digits that may be terminal figures: with each one of these combinations, within the terms of the question, the other six digits may be used as many times as there are combinations in the six taken together, that is $2\times3\times4\times5\times6=720$. Hence $119\times720=85,680$, the number required.

59. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

Find the sum of the mth power of all the numbers less than P and prime to it, and then by substitution find the sum when $m=1, 2, 3, 4, 5$.

Solution by the PROPOSER.

The sum of the mth powers of all numbers less than P and not prime to it is given by the sum of

$$\begin{aligned} & a^m+(2a)^m+(3a)^m+\ldots\ldots+[(P/a).a]^m \\ & +b^m+(2b)^m+(3b)^m+\ldots\ldots+[(P/b).b]^m \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & -(ab)^m-(2ab)^m-(3ab)^m-\ldots\ldots-[(P/ab).ab]^m \\ & -(bc)^m-(2bc)^m-(3bc)^m-\ldots\ldots-[(P/bc).bc]^m \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & +(abc)^m+(2abc)^m+(3abc)^m+\ldots\ldots+[(P/abc).abc]^m \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

$$=a^mS_{P/a}+b^mS_{P/b}+\ldots\ldots-(ab)^mS_{P/ab}-(bc)^mS_{P/bc}-\ldots\ldots+(abc)^mS_{P/abc}+\ldots\ldots$$

∴ The sum of the mth powers of all the integers less than P and prime to it is

$$S_P-a^mS_{P/a}+b^mS_{P/b}-\ldots+(ab)^mS_{P/ab}+(bc)^mS_{P/bc}+\ldots-(abc)^mS_{P/abc}-\ldots\ldots \quad (1).$$

But $$S_P=\frac{P^{m+1}}{m+1}+\tfrac{1}{2}P^m+B_1\frac{m}{2\,!}P^{m-1}+B_3\frac{m(m-1)(m-2)}{4\,!}P^{m-3}$$
$$+B_5\frac{m(m-1)(m-2)(m-3)(m-4)}{6\,!}P^{m-5}+\ldots\ldots$$

$$a^mS_{P/a}=\frac{P^{m+1}}{m+1}.(1/a)+\tfrac{1}{2}P^m+B_1\frac{m}{2\,!}P^{m-1}a+B_3\frac{m(m-1)(m-2)}{4\,!}P^{m-3}a^3$$
$$+B_5\frac{m(m-1)(m-2)(m-3)(m-4)}{6\,!}P^{m-5}a^5+\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Where $B_1=\frac{1}{6}$, $B_3=\frac{1}{30}$, $B_5=\frac{1}{42}$, $B_7=\frac{1}{30}$, etc., Bernoulli's numbers. Substituting the values of S_P, $a^mS_{P/a}$, $b^mS_{P/b}$, $(ab)^mS_{P/ab}$, etc., in (1) and letting S_m=the sum, we get

$$S_m=\frac{P^{m+1}}{m+1}\Big(1-(1/a)-(1/b)-(1/c)-\ldots\ldots+(1/ab)+\ldots\ldots\Big)$$
$$+\tfrac{1}{2}P^m\Big(1-n+\frac{n(n-1)}{2\,!}-\ldots\ldots\Big)+B_1\frac{m}{2\,!}P^{m-1}\{1-a-b-c-\ldots\ldots+ab+\ldots\ldots\}$$
$$-B_3\frac{m(m-1)(m-2)}{4\,!}P^{m-3}\{1-a^3-b^3-c^3-\ldots\ldots+a^3b^3+\ldots\ldots\}$$
$$+B_5\frac{m(m-1)(m-2)(m-3)(m-4)}{6\,!}P^{m-5}\{1-a^5-b^5-c^5-\ldots\ldots+a^5b^5+\ldots\ldots\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Where n is the number of primes in P. ∴ The coefficient of P^m is zero.

$$\therefore\ S_m=\frac{P^{m+1}}{m+1}[1-(1/a)][1-(1/b)][1-(1/c)]\ldots.$$

$$+B_1\frac{m}{2\,!}P^{m-1}(1-a)(1-b)(1-c)\ldots.$$

$$-B_3\frac{m(m-1)(m-2)}{4\,!}P^{m-3}(1-a^3)(1-b^3)(1-c^3)\ \ldots.$$

$$+B_5\frac{m(m-1)(m-2)(m-3)(m-4)}{6\,!}P^{m-5}(1-a^5)(1-b^5)(1-c^5)\ \ldots.$$

$$-\ldots\ldots+\ldots\ldots-\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

When $m=1$, $S_1=\frac{1}{2}P^2[1-(1/a)][1-(1/b)][1-(1/c)]\ldots.$

When $m=2$, $S_2=\frac{1}{3}P^3[1-(1/a)][1-(1/b)][1-(1/c)]..+\frac{1}{6}P(1-a)(1-b)(1-c)..$

When $m=3$, $S_3=\frac{1}{4}P^4[1-(1/a)][1-(1/b)][1-(1/c)]..+\frac{1}{4}P^2(1-a)(1-b)(1-c)..$

When $m=4$, $S_4=\frac{1}{5}P^5[1-(1/a)][1-(1/b)][1-(1/c)]..+\frac{1}{3}P^3(1-a)(1-b)(1-c)..$
$-\frac{1}{30}P(1-a^3)(1-b^3)(1-c^3)\ldots.$

When $m=5$, $S_5=\frac{1}{6}P^6[1-(1/a)][1-(1/b)][1-(1/c)]..+\frac{5}{12}P^4(1-a)(1-b)(1-c)..$
$-\frac{1}{12}P^2(1-a^3)(1-b^3)(1-c^3)\ldots.$

When $m=6$, $S_5=\frac{1}{7}P^7[1-(1/a)][1-(1/b)][1-(1/c)]..+\frac{1}{2}P^3(1-a)(1-b)(1-c)..$
$-\frac{1}{6}P^3(1-a^3)(1-b^3)(1-c^3)\ldots.\frac{1}{42}P(1-a^5)(1-b^5)(1-c^5)\ldots.$

PROBLEMS FOR SOLUTION.

ALGEBRA.

83. Proposed by J. MARCUS BOORMAN, Consultative Mechanician, Counselor at Law, Inventor, Etc., Woodmere, Long Island, N. Y.

Solve $x^2+y=8$......(1) ; $y^2+x=69$......(2), true to four decimals.

84. Proposed by BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, O.

On the present electoral basis, if all the electoral votes of each State are cast solid for one or the other of two presidential candidates, how many conbinations of States are possible for a total of 273 votes for the winning candidate ?

⁂ Solutions of these problems should be sent to J. M. Colaw, not later than May 10.

GEOMETRY.

91. Proposed by LEONARD E. DICKSON, Ph. D., Instructor in Mathematics in the University of California, Berkeley, Cal.

If a point A remain fixed while a point B moves along a given straight line, prove that the locus of the vertex C of the triangle ABC, similar to a given triangle and lying always on the same side of AB, is a straight line. Verify geometrically for the case in which the angles at A and C remain equal.